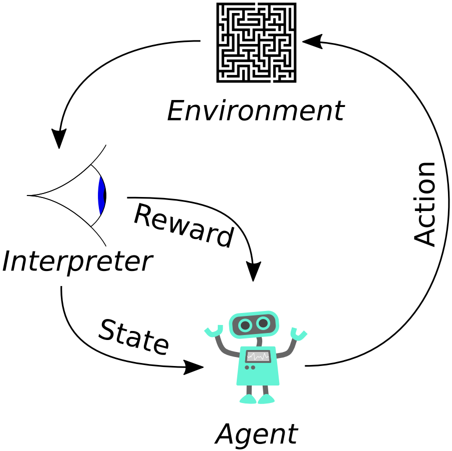
Deep he(a)p, big feat

arXiv:1707.06887 A Distributional Perspective on Reinforcement Learning
arXiv:1702.08165 Reinforcement Learning with Deep Energy-Based Policies

Reinforcement Learning



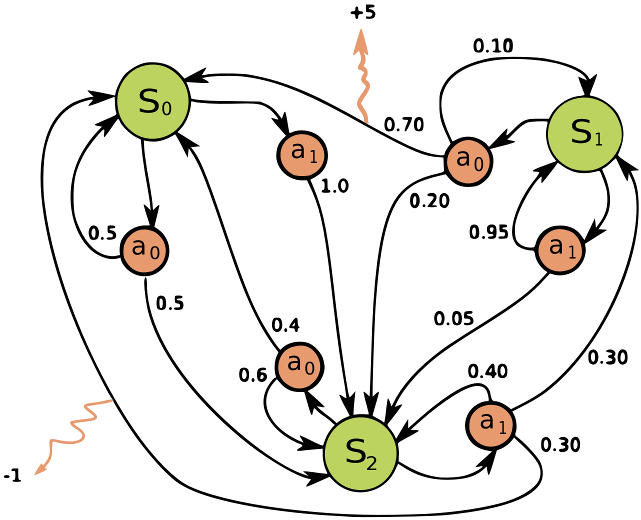
Reinforcement Learning

A framework for modeling intelligent agents. An agent takes an **action** depending on its **state** to change the **environment** with the goal of maximizing their **reward**.

Reinforcement Learning

- bandits / Markov decision process (MDP)
- episodes and discounts
- model-based RL / model-free RL
- single-agent / multi-agent
- tabular RL / Deep RL (parameterized policies)
- discrete / continuous
- on-policy / off-policy learning
- policy gradients / Q-learning

Markov decision process (MDP)



Markov decision process (MDP)

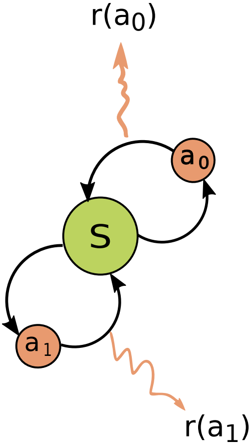
- states $s \in \mathcal{S}$
- actions $a \in \mathcal{A}$
- transition probability $p(s'|s, a)$
- rewards $r(s)$, $r(s, a)$, or $r(s, a, s')$

It's *Markov* because the transition $s_t \to s_{t+1}$ only depends on s_t.
It's a *decision process* because it depends on a.

Goal is to find policy $\pi(a|s)$ that maximizes reward over time.

Multi-armed bandits

Multi-armed bandits



Want to learn $p(r|a)$ and maximize $\langle r \rangle$. Tradeoff between *exploit* and *explore*.

Episodic RL

Agent either acts until a terminal state is reached.

$$
\begin{aligned}
s_0 &\sim \mu(s_0) \\
a_0 &\sim \pi(a_0|s_0) \\
r_0 &= r(s_0, a_0) \\
s_1 &\sim p(s_1|, s_0, a_0) \\
&\dots \\
a_{T-1} &\sim \pi(a_{T-1}|s_{T-1}) \\
r_{T-1} &= r(s_{T-1}, a_{T-1}) \\
s_T &\sim p(s_{T-1}|, s_{T-1}, a_{T-1})
\end{aligned}
$$

The goal is to maximize total rewards

$$\eta(\pi) = E[r_0 + r_1 + \cdots + r_{T-1}]$$

Discount factor

If there are no terminal states, the episode lasts "forever" and the agent takes "infinite" actions. In this case, we maximize discounted total rewards

$$\eta(\pi) = \eta(\pi) = E[r_0 + \gamma r_1 + \gamma^2 r_2 \cdots + \gamma^{T-1} r_{T-1}]$$

with discount $\gamma = [0, 1]$.

Without γ,

- the agent has no incentive to do anything *now*.
- η will diverge.

This means that the agent has an effective time-horizon

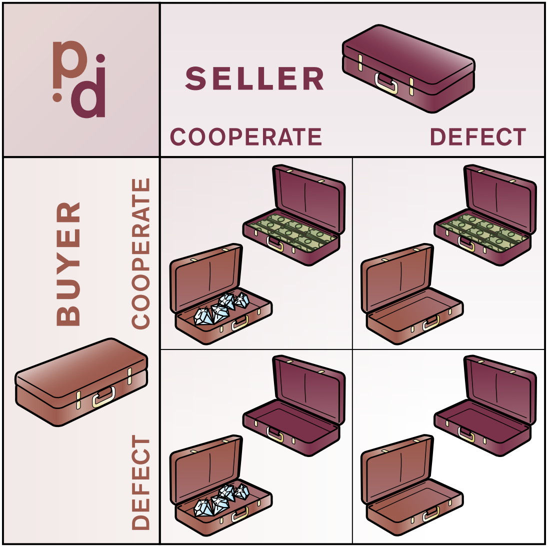
$$t_h \sim 1/(1-\gamma)$$

Model-based vs. Model-free

In model-based RL, we try to learn the transition function $p(s'|s, a)$. This let's us predict the expected next state $st + 1$ given state s_t and action a_t. *This means that the agent can think ahead and plan future actions.*

In model-free RL, we either try to learn $\pi(a|s)$ directly (policy gradient methods), or we learn a function $Q(s, a)$ that tells us the value of taking action a when in state s, which implies a $\pi(a|s)$. *This means that the agent has no "understanding" of the process and is essentially a lookup table.*

Multi-agent RL



Parameterized policies / Deep RL

If the total number of states is small, then Monte Carlo or dynamic programming techniques can be used to find $\pi(a|s)$ or $Q(s, a)$. These are sometimes referred to as *tabular methods*.

In many cases, this is intractable. Instead, we need to use a function approximator, such as a neural network, to represent these functions

$$\pi(a|s) \to \pi(a|s, \theta), \qquad Q(s, a) \to Q(s, a|\theta)$$

This takes advantage of the fact that in similar states we should take similar actions.

Discrete vs. continuous action spaces

Similarly, agents can either select from a discrete set of actions (i.e. left vs. right) or a continuum (steer the boat to heading 136 degrees). I'm not sure why people make a big deal out of the difference

- discrete: $\pi(a|s)$ is a discreete probability distribution.
- continuous: $\pi(a|s)$ is (just about always) Gaussian.

On-policy vs. off-policy

If our current best policy is $\pi(a|s)$, do we sample from $\pi(a|s)$ or do we sample from a different policy $\pi'(a|s)$?

- on-policy: Learn from $\pi(a|s)$, then update based on what worked well / didn't work well.
- off-policy: Learn from $\pi'(a|s)$ but update $\pi(a|s)$, letting us explore areas of state-action space that aren't likely to come up with our policy. *Can also learn from old experience*

Policy gradients

In which we just go for it and maximize the policy directly. Define

$$R[s(T), a(T)] \equiv \sum_{t=0}^{T} \gamma^t r(s(t), a(t))$$

We want to maximize $R(t)$, which depends on the trajectory

$$\begin{aligned}
\nabla_\theta \eta(\theta) &= \nabla_\theta \, E[R] \\
&= \nabla_\theta \, \sum p(R|\theta) \, R \\
&= \sum R \, \nabla_\theta p(R|\theta) \\
&= \sum R \, p(R|\theta) \, \nabla_\theta \log p(R|\theta) \\
&= E[R \, \nabla_\theta \log p(R|\theta)]
\end{aligned}$$

Policy gradients

The probability of a trajectory is

$$p(R|\theta) = \mu(s_0) \prod_{t=0}^{T-1} \pi(a_t|s_t, \theta)\, p(s_{t+1}|s_t, a_t)$$

which means that the derivative of it's log doesn't depend on the unknown transition function. This is model-free.

$$\nabla_\theta \log p(R|\theta) = \sum_{t=0}^{T-1} \nabla_\theta \log \pi(a_t|s_t)$$

$$\nabla_\theta \eta(\theta) = E\Big[R \sum_{t=0}^{T-1} \nabla_\theta \log \pi(a_t|s_t)\Big]$$

Policy gradients

Expressing the gradient as an expectation value means we can sample trajectories

$$E\left[R \sum_{t=0}^{T-1} \nabla_\theta \log \pi(a_t|s_t)\right] \rightarrow \frac{1}{N}\sum_{i=1}^{N}\left[R \sum_{t=0}^{T-1} \nabla_\theta \log \pi(a_t|s_t)\right]$$

and then do gradient descent on the policy

$$\theta \rightarrow \theta - \alpha \nabla_\theta \eta(\theta)$$

Since the gradient update is derived explicitly from trajectories sampled from $\pi(a|s)$, clearly this method is on-policy.

Policy gradients

$$
\begin{aligned}
\nabla_\theta \eta(\theta) &= E\Big[R \sum_{t=0}^{T-1} \nabla_\theta \log \pi(a_t|s_t)\Big] \\
&= E\Big[\sum_{t=0}^{T-1} \nabla_\theta \log \pi(a_t|s_t) \sum_{t'=t}^{T-1} \gamma^{t'-t} r(s_{t'}, a_{t'})\Big] \\
&= E\Big[\sum_{t=0}^{T-1} \nabla_\theta \log \pi(a_t|s_t)\, Q_\pi(s_t, a_t)\Big] \\
&= E\Big[\sum_{t=0}^{T-1} \nabla_\theta \log \pi(a_t|s_t)\, \Big(Q_\pi(s_t, a_t) - V_\pi(s_t)\Big)\Big]
\end{aligned}
$$

$$
Q_\pi(s_t, a_t) \equiv \sum_{t'=t}^{T-1} \gamma^{t'-t} r(s_{t'}, a_{t'}), \qquad V(s_t) \equiv \sum_{a_t} Q_\pi(s_t, a_t)\pi(a_t|s_t)
$$

Q-learning

What if we instead learn the Q-function or state-action value function associated with the optimal policy?

$$a_* = \arg\max_a Q_*(s, a)$$

Q-learning is model free

Just knowing the value function $V(s)$ of the state for a policy isn't enough to pick actions because we would need to know the transition function $p(s'|s, a)$.

Q-learning is off-policy

Expanding the definition of $Q(s_t, a_t)$, we see

$$Q_\pi(s_t, a_t) = E[r_t + \gamma V_\pi(s_{t+1})]$$
$$Q_\pi(s_t, a_t) = E\Big[r_t + \gamma E[Q_\pi(s_{t+1}, a_{t+1})]\Big]$$

This is known as *temporal difference learning*.

Now, let's find the optimal Q-function

$$Q_*(s_t, a_t) = E\Big[r_t + \gamma \max_a [Q_\pi(s_{t+1}, a)]\Big]$$

This is Q-learning.

DQN

If we have too many states, we instead minimize the loss

$$L(\theta) = \sum_t |r_t + \gamma \max_{a_{t+1}}[Q_\pi(s_{t+1}, a_{t+1}) - Q_\theta(s_t, a_t)|^2$$

via gradient descent

$$\theta \to \theta - \alpha \nabla_\theta L(\theta)$$

Q learning II, the SQL

Define

$$\operatorname{soft}\max_x f(x) \equiv \log \int dx\, e^{f(x)}$$

Then, soft Q-learning is

$$Q_*(s_t, a_t) = E[r_t + \gamma \operatorname{soft}\max_a Q(s_{t+1}, a)]$$

which has optimal policy

$$\pi(a|s) \propto \exp Q(s, t).$$

Trade-off between optimality and entropy. Allows transfer learning by letting policies compose.

A Distributional Perspective on Reinforcement Learning

Learn a distribution over Q-values. Let $Z(s_t, a_t)$ have an expectation value that is $Q(s, a)$. Then we learn

$$Z(s_t, a_t) = r_t + \gamma Z(s_{t+1}, a_{t+1})$$